

JEE: WAVE

1. A block attached to a spring executes SHM on a rough horizontal surface with coefficient of friction μ . Initial amplitude is A . Find amplitude after one oscillation.

ANSWER

Friction $\Rightarrow F = \mu mg$

Distance in one full oscillation = $4A$

Work done,

$$W = F \cdot 4A = 4\mu mgA$$

Energy loss

Initial, $E_i = \frac{1}{2} kA^2$

Final, $E_f = \frac{1}{2} kA'^2$

Energy lost = work done

$$\frac{1}{2} kA^2 - \frac{1}{2} kA'^2 = 4\mu mgA$$

$$A^2 - A'^2 = \frac{8\mu mgA}{k}$$

$$A - A' = \frac{4\mu mg}{k}$$

$$A' = A - \frac{4\mu mg}{k}$$

2. A mass m is attached between 2 springs of constants k_1 & k_2 fixed at two walls. Find the time period

ANSWER

Net force

$$F = -(k_1 + k_2)x$$

$$m\ddot{x} = -(k_1 + k_2)x$$

Angular frequency

$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$

3. A block of mass m is attached to a spring of force constant k . The other end of the spring is attached to a support that moves with constant acc. a to the right. Find time period.

ANSWER

$$F_{\text{pseudo}} = -ma$$

At equilibrium,

$$kx_0 = ma$$

$$F = -ky$$

$$m\ddot{y} = ky$$

$$\omega = \sqrt{\frac{k}{m}}$$

Final answer

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Q The equation of a wave travelling on a string is $y = \sin [20\pi x + 10\pi t]$, where x and t are distance and time in SI units. The minimum distance between two points having the same oscillating speed is:

Sol: $y = \sin [20\pi x + 10\pi t]$

We know distance is half the wavelength ($\frac{\lambda}{2}$)

$$\lambda = \frac{2\pi}{k}, \text{ wave no. } k = 20\pi$$

$$\lambda = \frac{2\pi}{20\pi} = \frac{1}{10} \text{ m} = 10 \text{ cm}$$

Thus min. distance between two points having the same speed is

$$\text{Distance} = \frac{\lambda}{2} = \frac{10}{2} = 5 \text{ cm}$$

Q Two harmonic waves moving in the same direction superimpose to form a wave $x = a \cos(1.5t) \cos(50.5t)$ where t is in seconds. Find the period with which they beat. (Close to nearest integer)

Sol: $x = a \cos(1.5t) \cos(50.5t)$

by using product of cosines identity

$$x = \frac{a}{2} \cos[(1.5 + 50.5)t] + \frac{a}{2} \cos[(50.5 - 1.5)t]$$

$$x = \frac{a}{2} \cos(52t) + \frac{a}{2} \cos(49t)$$

This represents two waves with angular frequencies 52 and 49

beat frequency = difference in frequencies

$$f_1 = \frac{52}{2\pi}, f_2 = \frac{49}{2\pi}, f_{\text{beat}} = f_1 - f_2 = \frac{3}{2\pi} \text{ Hz}$$

$$T_{\text{beat}} = \frac{1}{f_{\text{beat}}} \Rightarrow T_{\text{beat}} = \frac{2\pi}{3} \text{ sec}$$

$$\therefore T_{\text{beat}} \approx 2.09 \text{ sec} \approx 2 \text{ sec}$$

Q Displacement of a wave is expressed as $x(t) = 5 \cos(628t + \frac{\pi}{2})$ m. The wavelength of the wave when its velocity is 300 m/s is: ($\pi = 3.14$)

Sol: $x(t) = 5 \cos[628t + \frac{\pi}{2}]$ m

velocity (v_w) = 300 m/s

$$v_w = \frac{\omega}{k}$$

$$300 = \frac{628}{k} \Rightarrow k = \frac{628}{300}$$

$$\frac{2\pi}{\lambda} = \frac{628}{300} \Rightarrow \lambda = \frac{2 \times 3.14 \times 300}{628}$$

$$\lambda = \underline{\underline{2.3\text{ m}}}$$

JEE Questions Waves

Pranav Sardhar
Batch - 2025-26

①

Q.1) The velocity of sound in air is doubled when the temperature is raised from 0°C to $\alpha^\circ\text{C}$. The value of α is —. [JEE Main 2026]

Ans: ~~⊗~~ $V = \sqrt{\frac{\gamma RT}{M}}$ {Formula}

$$V \propto \sqrt{T}$$

$$T_1 = 0^\circ\text{C} \Rightarrow 273\text{K}$$

$$V_1 = V$$

$$T_2 = \alpha^\circ\text{C} \Rightarrow (\alpha + 273)\text{K}$$

$$V_2 = 2V_1$$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}$$

$$4 = \frac{\alpha + 273}{273}$$

$$\alpha = 1092 - 273$$

$$\therefore \alpha \Rightarrow 819$$

Q.2) A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is :- [JEE Main 2024]

Ans: ~~⊗~~ Overtone:- Resonant frequency higher than the fundamental frequency.

$$f_o = \frac{V}{2L} \text{ {Open organ pipe}}$$

$$f_c = \frac{V}{4L} \text{ {Closed organ pipe}}$$

Seventh Overtone $\Rightarrow$ 8th harmonic

$$f_0 = 8 \times \frac{v}{2L}$$

(2)

Odd harmonics for closed pipe. So;

$$f_c = (2(7)+1) \times \frac{v}{4L}$$

$$f_c = 15 \times \frac{v}{4L}$$

$$\frac{f_c}{f_0} = \frac{15 \cancel{v}}{4 \cancel{L}} \times \frac{2 \cancel{L}}{8 \cancel{v}}$$

$$\frac{f_c}{f_0} = \frac{15}{16} \Rightarrow \frac{a-1}{a}$$

$$\boxed{\therefore a = 16}$$

Q.3) A sonometer wire of resonating length 90cm has a fundamental frequency of 400Hz when kept under some tension. The resonating length of the wire with fundamental frequency of 600Hz under same tension — cm. [JEE Main 2024]

Ans: (*) $f = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$ { Formula, where $\mu \Rightarrow$ Linear mass density }

$$400 = \frac{1}{2 \times 0.9} \sqrt{\frac{T}{\mu}} \quad \text{--- (1)}$$

$$600 = \frac{1}{2L_2} \sqrt{\frac{T}{\mu}} \quad \text{--- (2)}$$

$$\Rightarrow \frac{(1)}{(2)} = \frac{4}{6} = \frac{L_2}{0.9}$$

$$L_2 \Rightarrow \frac{2 \times 0.9}{3} \Rightarrow 0.6 \text{ metres}$$

$$\boxed{\therefore L_2 \Rightarrow 60 \text{ cm}}$$

Waves

Q.1. A uniform rope of length L and mass m hangs vertically from a rigid support. A block of mass M is attached to the free end of the rope. A transverse pulse is produced at the lower end of the rope. What is the time taken by the pulse to reach the top of the rope?

Ans. The tension T at height x ,

$$T = Mg + \left(\frac{m}{L}x\right)g$$

Velocity (v) at any point x , $\left(\mu = \frac{m}{L}\right)$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\begin{aligned} v &= \sqrt{\frac{(M + m/L x)g}{m/L}} \\ &= \sqrt{\frac{mLg}{m} + xg} \end{aligned}$$

$v = \frac{dx}{dt}$, $dt = \frac{dx}{v}$, integrating from 0 to L ,

$$t = \int_0^L \frac{dx}{\sqrt{\frac{mLg}{m} + xg}}$$

$$\therefore t = \frac{2}{\sqrt{g}} \left[\sqrt{\frac{mLg}{m} + xg} - \sqrt{\frac{mLg}{m}} \right]$$

Q2. A string fixed at both ends is vibrating in its 4th harmonic. The equation of the standing wave is $y = 0.3 \sin(0.2x) \cos(600t)$. What is the length of the string? (Assume x is in cm and t is in seconds)

Ans. $k = 0.2 \text{ cm}^{-1}$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi \text{ cm}$$

For 4th harmonic ($n=4$):

$$L = 4 \left(\frac{\lambda}{2} \right) = 2\lambda$$

$$\therefore L = 2(10\pi) = 20\pi \approx \underline{\underline{62.8 \text{ cm.}}}$$

Q3. A whistle revolving in a circle of radius 50 cm at an angular speed of 20 rad/s emits a sound of frequency 400 Hz. What is the maximum and minimum frequency ~~hear~~ heard by an observer located far away from the center of the circle? (Speed of sound $v = 340 \text{ m/s}$).

Ans. $V_s = r\omega = 0.5 \text{ m} \times 20 \text{ rad/s} = 10 \text{ m/s}$

$$f_{\text{max}} = f_0 \left(\frac{V}{V - V_s} \right) = 400 \left[\frac{340}{340 - 10} \right] = \underline{\underline{412.12 \text{ Hz}}}$$

$$f_{\text{min}} = f_0 \left(\frac{V}{V + V_s} \right) = 400 \left[\frac{340}{340 + 10} \right] = \underline{\underline{388.57 \text{ Hz}}}$$

Name: Dhara K.P

Class: XI H

Roll No.: 12

WAVES

Q1. A tuning fork resonates with a sonometer wire of length 1m stretched with a tension of 6N. When the tension in wire is changed to 54N, same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is — Hz.

Ans. $f = \frac{1}{2L} \sqrt{\frac{T}{\rho}}$

$$f_1 = \frac{1}{2L} \sqrt{\frac{T_1}{\rho_1}} \Rightarrow \underline{f_1 = \frac{1}{2} \sqrt{\frac{6}{\rho_1}}}$$

$$\underline{f_2} = \frac{1}{2L} \sqrt{\frac{T_2}{\rho_2}} \Rightarrow \underline{f_2} = \frac{1}{2} \sqrt{\frac{54}{\rho_2}} = \frac{3}{2} \sqrt{\frac{6}{\rho_2}} = \underline{3f_1}$$

Since the ~~2nd~~ 2nd instance of string produces 12 beats per second, this means that the frequency of the tuning fork (a original string) Δ the new frequency (of the string with higher tension) differ by 12 Hz. If f_F is frequency of tuning fork. Then:

$$\Rightarrow \text{Either } f_2 = f_F + 12 \text{ Hz} \quad \text{or} \quad f_2 = f_F - 12 \text{ Hz}$$

Since we know, $f_2 = 3f_1$ and $f_1 = 3f_F$,

$$3f_F = f_F + 12 \quad \text{or} \quad 3f_F = f_F - 12$$

$$\Rightarrow 3f_F - f_F = -12 \text{ Hz}$$

$$\Rightarrow 2f_F = -12$$

This isn't possible

$$\therefore f_2 = f_F + 12$$

$$\Rightarrow 2f_F = 12$$

$$\Rightarrow \underline{f_F = 6 \text{ Hz}}$$

$\therefore$ The frequency of tuning fork 6 Hz.

Q2. A closed organ pipe of 150cm long gives 7 beats per second with an open organ pipe of length 350cm, both vibrating in fundamental mode. The velocity of sound is ____ m/s.

Ans. $f_c = \frac{v}{4l_1}$ $f_o = \frac{v}{2l_2}$

$$|f_c - f_o| = 7$$

$$\Rightarrow \frac{v}{600\text{cm}} - \frac{v}{700\text{cm}} = 7$$

$$\Rightarrow \frac{v}{6\text{m}} - \frac{v}{7\text{m}} = 7$$

$$\Rightarrow v = 42 \times 7 = \underline{\underline{294\text{m/s}}}$$

Q3. The displacement 'y' of a wave travelling in the x-direction is given by $y = 10^{-4} \sin(600t - 2x + \pi/3)\text{m}$, where x is expressed in metre & t in second. The speed of the wave-motion is m/s is $\rightarrow$

Ans. $y = a \sin(\omega t - kx + \phi)\text{m}$

$$y = 10^{-4} \sin(600t - 2x + \pi/3)$$

$$\underline{\omega = 600\text{sec}^{-1}} ; \underline{k = 2\text{m}^{-1}} ; \frac{\omega}{k} = \frac{2\pi b}{2\pi/\lambda} = \underline{\underline{b\lambda = \text{Velocity}}}$$

$$\therefore \text{Velocity} = \frac{\omega}{k} = \frac{600}{2} = \underline{\underline{300\text{m/s}}}$$

Waves

Q) Two identical string X and Z made of same material have tension T_x and T_z in them. If their fundamental frequencies are 450Hz & 300Hz , ratio T_x/T_z is:

- (A) 2.25 (B) 0.44 (C) 1.25 (D) 1.5

Sol.

$$f = \frac{1}{2l} \sqrt{\frac{T}{\mu}} \Rightarrow T = 4f^2 l^2 \mu$$

$$T_x = 4f_x^2 l^2 \mu \text{ \& } T_z = 4f_z^2 l^2 \mu \left(\begin{array}{l} \text{same length \& } \mu \\ f_x = 450 \text{ \& } f_z = 300\text{Hz} \end{array} \right)$$

$$\frac{T_x}{T_z} = \frac{4f_x^2 l^2 \mu}{4f_z^2 l^2 \mu} = \frac{f_x^2}{f_z^2} = \frac{450 \times 450}{300 \times 300} = \frac{9}{4} = \underline{\underline{2.25}}$$

$$\boxed{\frac{T_x}{T_z} = 2.25}$$

$$\boxed{(A) 2.25}$$

Q) A string is clamped at both ends and it is vibrating in its 4th harmonic. The equation of stationary wave is $y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of string is:

- (A) 60m (B) 20m (C) 80m (D) 40m

Sol.

For 4th harmonic:

$$l = \frac{4\lambda}{2} = 2\lambda$$

$$\frac{2\pi}{\lambda} = 0.157$$

$$\lambda = \frac{2 \times 3.14}{0.157} = 40$$

$$l = 2\lambda = 2 \times 40 = 80\text{m}$$

$$\boxed{(C) 80\text{m}}$$

Q) The amplitude and phase of a wave that is formed by superposition of two harmonic travelling waves, $y_1(x,t) = 4 \sin(kx - \omega t)$ and $y_2(x,t) = 2 \sin(kx - \omega t + \frac{2\pi}{3})$ are:
(Take angular frequency of initial waves same as ω)

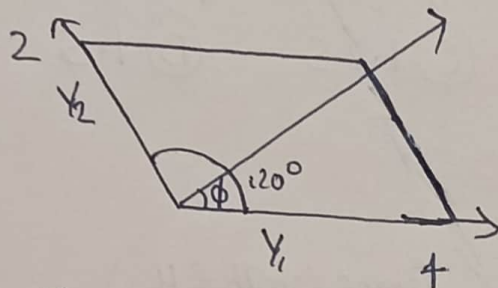
(A) $[\sqrt{3}, \frac{\pi}{6}]$

(B) $[2\sqrt{3}, \frac{\pi}{6}]$

(C) $[6, \frac{2\pi}{3}]$

(D) $[6, \frac{2\pi}{3}]$

Sol.



$$y_1 = 4 \sin(kx - \omega t), A_1 = 4$$

$$y_2 = 2 \sin(kx - \omega t + \frac{2\pi}{3}), A_2 = 2$$

(120°) $\frac{2\pi}{3}$ in y_2 eq. is angle between $\vec{y}_1$ and $\vec{y}_2$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \theta} \quad (\text{Parallelogram law of vector})$$

$$A = \sqrt{16 + 4 + 2 \times 4 \times 2 \times \cos 120}$$

$$A = \sqrt{20 + 16 \times -\frac{1}{2}}$$

$$A = \sqrt{12} = 2\sqrt{3}$$

$$\tan \phi = \frac{A_2 \sin \theta}{A_1 + A_2 \cos \theta} \quad \left(\begin{array}{l} \text{Since, for A \& B vector} \end{array} \Rightarrow \tan \theta = \frac{B \sin \theta}{A + B \cos \theta} \right)$$

$$\tan \phi = \frac{2 \times \frac{\sqrt{3}}{2}}{4 + 2 \times -\frac{1}{2}} = \frac{\sqrt{3}}{4-1} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\text{So } \phi = \frac{\pi}{6}$$

$$\text{Thus, } A = 2\sqrt{3} \text{ \& } \phi = \frac{\pi}{6}$$

(B) $[2\sqrt{3}, \frac{\pi}{6}]$

WAVES

① The fifth harmonics of a closed organ pipe is found to be in unison with first harmonic of an open pipe is $5/\alpha$. The value of α is 2.

Ans:-

$$f_n = \frac{n v}{4L_c}, n = 1, 3, 5, 7, \dots \quad \left| \quad f_c = \frac{5v}{2L_o}, n = 1, 2, 3, \dots$$

$$f_{\text{open}} = \frac{v}{2L_o} \quad \left| \quad \frac{5v}{4L_c} = \frac{v}{2L_o} \Rightarrow 10L_o = 4L_c \Rightarrow \frac{L_c}{L_o} = 5/2.$$

$$\therefore \frac{5}{\alpha} = \frac{5}{2} \Rightarrow \alpha = 2.$$

② In an open organ pipe v_3 and v_6 are 3rd and 6th harmonic frequencies, respectively. If $v_6 - v_3 = 2200 \text{ Hz}$ then length of the pipe is 225 mm. ($v_s = 330 \text{ m/s}$)

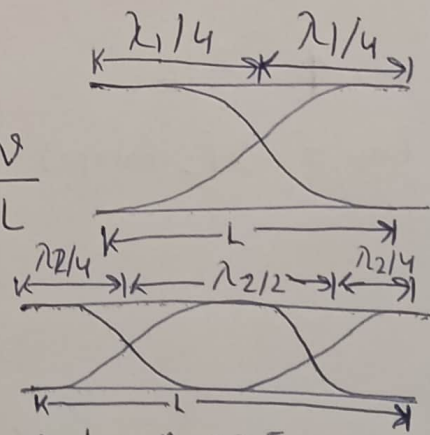
Ans:-

$$v_n = \frac{n v}{2L} \quad \left| \quad v_3 = \frac{3v}{2L} \quad ; \quad v_6 = \frac{6v}{2L}$$

$$\therefore v_6 - v_3 = 2200$$

$$\Rightarrow \frac{3v}{2L} = 2200 \Rightarrow L = \frac{2200 \times 2}{3 \times 330}$$

$$\Rightarrow L = 0.225 \text{ m} = 225 \text{ mm}$$



③ The amplitude and phase of a wave that is formed by the superposition of two harmonic travelling waves, $y_1(x,t) = 4\sin(kx - \omega t)$ and $y_2(x,t) = 2\sin(kx - \omega t + \frac{2\pi}{3})$ are: $[2\sqrt{3}, \pi/6]$.

Ans:-

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

Given,

$$\left[A_1 = 4, A_2 = 2, \phi = 120^\circ \right]$$

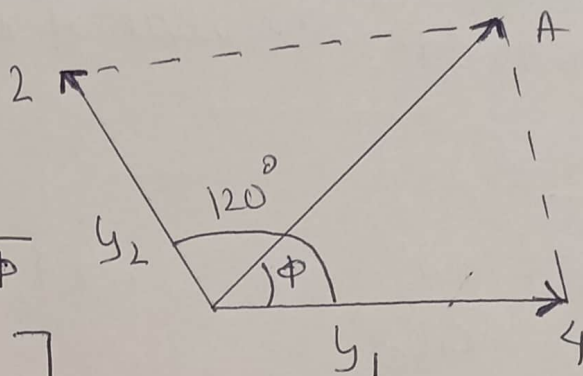
$$= \sqrt{4^2 + 2^2 + 2 \times 4 \times 2 \times \cos(120^\circ)} = \sqrt{16 + 4 - 8}$$

$$= \sqrt{12} = 2\sqrt{3}$$

$$\therefore \tan \phi = \frac{A_2 \sin(\phi)}{A_1 + A_2 \cos(\phi)} = \frac{2 \sin 120^\circ}{4 + 2 \cos 120^\circ} = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \phi = \pi/6$$

Ans:- $2\sqrt{3}$ (amp.) ; $\phi = \pi/6$ (phase)



WAVES

(Q) Two tuning fork A and B are sounded together giving rise to 8 beats in 2 seconds. When fork A is loaded with wax, the beat frequency reduces to 4 beats in 2 seconds. If the frequency of fork B is 380 Hz, then the original frequency of fork A is:

- (A) 376 Hz (B) 384 Hz (C) 388 Hz (D) 372 Hz

Solution

$$\text{Beat frequency} = |f_A - f_B|$$

$$\text{Initially } |f_A - 380| = 4$$

$$\text{So } f_A = 376 \text{ or } 384$$

When fork A is loaded, its frequency decreases, Beat frequency decreases $\rightarrow$ fork A must have been higher initially

$$\text{Thus, } f_A = 384 \text{ Hz option (B)}$$

(Q) The velocity of sound in air is doubled when the temperature is raised from 0° to x° C. The value of x is

- (A) 819° C (B) 546° C (C) 273° C (D) 1092° C

Solution

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = 2$$

$$\frac{T_2}{T_1} = 4$$

$$\text{at } 0^\circ \text{C } T = 273 \text{ K}$$

$$T_2 = 4 \times 273 = 1092 \text{ K}$$

$$x = 1092 - 273 = 819^\circ \text{C (option A)}$$

(Q) In a stationary wave, the distance between a node and next antinode is

(A) λ (B) $\frac{\lambda}{2}$ (C) $\frac{\lambda}{4}$ (D) $\frac{\lambda}{8}$

Solution

Node to Antinode separation

$$= \frac{\lambda}{4}$$

Option (C)

Waves

Q.1 A string is stretched between fixed points separated by 75 cm . It is observed to have resonant frequencies of 420 Hz and 315 Hz . There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is.

$$\text{Ans: } 315 = nv / 2L \rightarrow (1)$$

$$420 = (n+1)v / 2L \rightarrow (2)$$

Divide (1) by (2)

$$n=3$$

$$f_0 = v / 2L = 315 / 3 = 105\text{ Hz}$$

Q.2 A wave travelling along the x -axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m

and 2 s resp, then α and β in appropriate unit are

Ans: $y(x, t) = 0.005 \cos(\alpha x - \beta t)$

$A = k = \frac{2\pi}{\lambda}$ as $\lambda = 0.08\text{m}$

$a = \frac{2\pi}{0.08} = \frac{\pi}{0.04} = a = \frac{\pi}{4} \times 100 = 25\pi$

$\omega = \beta = \frac{2\pi}{2} = \pi$

$\alpha = 25\pi, \beta = \pi$

Q.3 A uniform string of length 20m is suspended from a rigid support. A short moving up the string. The time taken to reach the support is?

Ans: $\mu = \frac{m}{L}$

$v = \sqrt{\frac{T}{\mu}}$

$\frac{dx}{dt} \sqrt{\frac{mgx/L}{m/L}} = \sqrt{g} \sqrt{x}$

$\frac{dx}{\sqrt{x}} = \sqrt{g} dt$

$\int_0^L x^{-1/2} dx = \sqrt{g} \int_0^L dt$

$t = 2\sqrt{L/g} = 2 \times \sqrt{20/10} = 2\sqrt{2} \text{ s}$

Waves

Zaid ShamShæri

XI - A .

2025-2026

Q. $y_1(x,t) = 4 \sin(kx - \omega t)$

$$y_2(x,t) = 2 \sin(kx - \omega t + 2\frac{\pi}{3})$$

Find amplitude and phase .

Sol :

$$A = \sqrt{4^2 + 2^2 + 2 \times 4 \cdot 2 \cos(2\frac{\pi}{3})}$$

$$\cos(2\frac{\pi}{3}) = -\frac{1}{2}$$

$$A = \sqrt{16 + 4 - 8} = \sqrt{12} = 2\sqrt{3}$$

Phase ϕ

$$\tan \phi = \frac{2 \sin(2\pi/3)}{4 + 2 \cos(2\pi/3)}$$

$$\Rightarrow \frac{2(\sqrt{3}/2)}{4-1} = \frac{\sqrt{3}}{3}$$

$$\phi = \frac{\pi}{6}$$

Q2 $x(t) = 5 \cos(628t + \frac{\pi}{2}) \text{ m}$.

If wave velocity is 300 m find wavelength.

Sol:

$$x(t) = A \cos(2\pi f t + \phi)$$

$$\omega = 628 = 2\pi f \Rightarrow f = \frac{628}{2\pi} = 100 \text{ Hz}$$

$$\lambda = \frac{v}{f} = \frac{300}{100} = 3 \text{ m}$$

Q3 . Transvers wave .

$$y(x, t) = 4.0 \sin(20 \times 10^{-3} x + 600 t) \text{ mm}$$

find wave velocity

Sol:

$$y = A \sin(kx \pm \omega t)$$

$$k = 20 \times 10^{-3} \text{ (mm}^{-1}\text{)} , \omega = 600 \text{ s}^{-1}$$

$$k = 20 \times 10^{-3} \text{ mm} = 20 \text{ m}^{-1}$$

$$v = \frac{\omega}{k} = \frac{600}{20} = 30 \text{ m/s}$$

In an experiment with sonometer when a mass of 180 g is attached to the string, it vibrates with fundamental frequency of 30 Hz. When a mass m is attached, the string vibrates with fundamental frequency of 50 Hz. The value of m is — g.

[JEE Main 2023]

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$$

Mass attached to the string is 180 g & frequency is 30 Hz :

$$\frac{f_2}{30 \text{ Hz}} = \sqrt{\frac{T_2}{T_1}}$$

$$\text{Frequency is 50 Hz : } \frac{50 \text{ Hz}}{30 \text{ Hz}} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{5}{3} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{25}{9} = \frac{T_2}{T_1}$$

$$\frac{m}{180 \text{ g}} = \frac{T_2}{T_1} = \frac{25}{9}$$

$$m = \frac{25}{9} \times 180 = \underline{\underline{500 \text{ g}}}$$

A guitar string of length 90 cm vibrates with a fundamental frequency of ~~string producing~~ ^{120 Hz}. The length of string producing a fundamental frequency of 180 Hz will be — cm. [JEE Main 2020]

Solution:

$$f = v/2L$$

Eq for fundamental frequency of original string & shorter string:

$$f_1 = v/2L_1 \quad f_2 = v/2L_2$$

$$f_1 = 120 \text{ Hz} \quad L_1 = 90 \text{ cm (original string)}$$

$$f_2 = 180 \text{ Hz} \quad L_2 = ? \text{ (shorter string)}$$

$$\frac{f_1}{f_2} = \frac{L_2}{L_1}$$

$$\frac{120 \text{ Hz}}{180 \text{ Hz}} = \frac{90 \text{ cm}}{L_2}$$

$$L_2 = \frac{120}{180} \times 90 = \underline{\underline{60 \text{ cm}}}$$

A transverse harmonic wave on a string is given by $y(x, t) = 5 \sin(6t + 0.003x)$ where x & y are in cm and t in sec. The wave velocity is — ms^{-1} . [JEE Main 2023]

Solution:

The general eq for transverse harmonic wave:

$$y(x, t) = A \sin(kx - \omega t + \phi)$$

$$v = \frac{\omega}{k}$$

$$A = 5 \text{ cm} \quad k = 0.003 \text{ cm}^{-1} \quad \omega = 6 \text{ rad/s}$$

$$\therefore v = \frac{\omega}{k} = \frac{6}{0.003} = 2000 \text{ cm/s} \\ = \underline{\underline{20 \text{ m/s}}}$$

Q.) The Velocity of Sound in air is doubled when T is raised from 0° to 2° C

The value of ' 2 ' is _____

Ans: $V \propto \sqrt{T}$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}} = \boxed{\frac{T_2}{T_1} = 4}$$

$$T_1 = 0^\circ \text{C}, = 273.15 \text{ K}$$

$$T_2 = 273.15 + 2 \text{ K}$$

$$\boxed{2 = 819^\circ \text{C}}$$

Q.) Displacement of wave is expressed as ~~$x(t) = 5 \cos$~~
 $x(t) = 5 \cos \left(628t + \frac{\pi}{2} \right) \text{ m}$. The ' λ ' of wave
 ~~x~~ when its velocity is 300 ms^{-1} is _____ m

Ans

$$X(t) = 5 \cos \left(628t + \frac{\pi}{2} \right) \text{ m}$$

$$v = 300 \text{ ms}^{-1}$$

$$k = \frac{628}{300}$$

$$\frac{2\pi}{\lambda} = \frac{628}{300}$$

$$\boxed{\lambda = 2 \text{ m}}$$

Q.) In an experiment with closed organ pipe.
It's filled with water by $(\frac{1}{5})^{\text{th}}$ of its volume.
The frequency of the fundamental note will change by

(A) 25%.

(C) 20%.

(B) - 20%.

(D) - 25%.

Ans $\rightarrow$

$$\lambda_1 = 4l$$

$$f_1 = \frac{v}{4l}$$

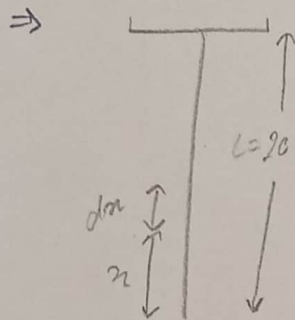
$$\lambda_2 = \frac{16l}{5}$$

$$f_2 = \frac{5v}{16l}$$

$$\frac{\Delta f}{f} = \frac{\frac{v}{l} \cdot \frac{1}{16}}{\frac{v}{4l}} \times 100 = \boxed{25\%}$$

Waves

Q A uniform string of length 20m is suspended from rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (g = 10 m/s²)



$$\mu = \frac{m}{L}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\frac{dx}{dt} = \sqrt{\frac{mgx/L}{m/L}} = \sqrt{g} \sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = \sqrt{g} dt$$

$$\int_0^L x^{-1/2} dx = \sqrt{g} \int_0^L dt$$

$$t = \frac{2\sqrt{L}}{\sqrt{g}} = 2 \times \sqrt{\frac{20}{10}} \Rightarrow \boxed{t = 2\sqrt{2} \text{ s}} \rightarrow \text{Ans}_2$$

Q A student performing experiment of Resonance Column. The diameter of the column tube is 4cm. The frequency of tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of water level in the column is:

$$\Rightarrow e = (0.6)r = (0.6)2$$

$$= 1.2 \text{ cm}$$

$$\frac{v}{4}(1+e) = f$$

$$1+e = \frac{v}{4f}$$

$$L = \frac{v}{4f} - e$$

$$= \left(\frac{336 \times 10^2}{4 \times 512} \right) - 1.2$$

$$\boxed{L = 15.2 \text{ cm}} \quad \text{--- Ans.}$$

Q. A stationary source emits sound of frequency ~~492~~ $f_0 = 492 \text{ Hz}$. The sound is reflected by a large car approaching the source with a speed of 2 m/s . The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Speed of sound in air is 330 m/s)

$$\Rightarrow f_1 = \left(\frac{c + v_0}{c} \right) f_0 = \left(\frac{330 + 2}{330} \right) 492 \text{ Hz} = 494.98 \text{ Hz}$$

$$\therefore f_{\text{reflected (car)}} = f_1$$

$$f_2 = \left(\frac{c}{c - v_0} \right) f_{\text{ref}} = \left(\frac{330}{330 - 2} \right) f_1 = \frac{330}{328} \times \frac{332}{330} \times 492$$

$$\therefore \text{Beat } f = |f_0 - f_2| = \left(\frac{332}{328} - 1 \right) 492$$

$$= \boxed{16 \text{ Hz}} \quad \text{--- Ans.}$$

Q1) The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(m) \sin 2\pi \left(\frac{t}{0.04(s)} - \frac{x}{0.5(m)} \right)$$

Calculate tension in string.

A) Linear mass density $\mu = 0.04 \text{ kg m}^{-1}$

$$y = A \sin(\omega t - kx)$$

$$\omega = \frac{2\pi}{0.04} \text{ rad s}^{-1}, \quad k = \frac{2\pi}{0.5} \text{ rad m}^{-1}$$

$$v = \frac{\omega}{k} = \frac{2\pi}{0.04} \times \frac{0.5}{2\pi}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$$

$$T = \mu \omega^2 / k^2$$

$$= (0.04 \times (2\pi / 0.04)^2) / (2\pi / 0.5)^2$$

$$= 6.25 \text{ N}$$

Q2) A uniform strength string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is. (take $g = 10 \text{ m s}^{-2}$)

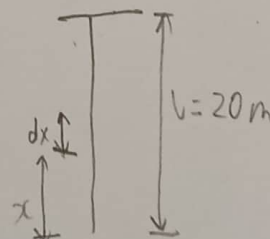
A) $\mu = m/L$

As, $v = \sqrt{\frac{T}{\mu}}$

$$\frac{dx}{dt} = \sqrt{\frac{mgx/L}{m/L}} = \sqrt{gx}$$

$$\frac{dx}{\sqrt{x}} = \sqrt{g} dt$$

$$\int_0^L x^{-1/2} dx = \sqrt{g} \int_0^t dt$$



$$t = \frac{2\sqrt{L}}{\sqrt{g}} = 2\sqrt{\frac{20}{10}} = 2\sqrt{2} \text{ s} //$$

Time period = $2\sqrt{2}$ seconds

Q3) A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if the density and elasticity of steel are $7.7 \times 10^3 \text{ kg/m}^3$ and $2.2 \times 10^{11} \text{ N/m}^2$ respectively?

$$\begin{aligned} \text{A) } f &= \frac{v}{2L} & \gamma &= \frac{TL}{A\Delta L} \\ &= \frac{1}{2L} \sqrt{\frac{T}{\mu}} \\ &= \frac{1}{2L} \sqrt{\frac{T}{Ad}} \end{aligned}$$

$$T/A = \frac{\gamma \Delta L}{L}$$

$$f = \frac{1}{2L} = \sqrt{\frac{\gamma \Delta L}{Ld}}$$

$$L = 1.5 \text{ m}, \Delta L/L = 0.01$$

$$d = 7.7 \times 10^3 \text{ kg/m}^3$$

$$\gamma = 2.2 \times 10^{11} \text{ N/m}^2$$

$$f = \frac{1}{2L} \sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}} = 178.2 \text{ Hz} //$$